CHAPTER **8**

Financial Options and Applications in Corporate Finance

In 2008, Cisco had almost 1.2 billion outstanding employee stock options and about 5.9 billion outstanding shares of stock. If all these options are exercised, then the option holders will own 16.9% of Cisco's stock: 1.2/(5.9+1.2) = 0.169. Many of these options never may be exercised, but any way you look at it, 1.2 billion is a lot of options. Cisco isn't the only company with mega-grants: Pfizer, Time Warner, Ford, and Bank of America are among the many companies that have granted to their employees options to buy more than 100 million shares. Whether your next job is with a high-tech firm, a financial service company, or a manufacturer, you will probably receive stock options, so it's important that you understand them.

In a typical grant, you receive options allowing you to purchase shares of stock at a fixed price, called the strike price or exercise price, on or before a stated expiration date. Most plans have a vesting period, during which you can't exercise the options. For example, suppose you are granted 1,000 options with a strike price of $50, an expiration date 10 years from now, and a vesting period of 3 years. Even if the stock price rises above $50 during the first 3 years, you can't exercise the options because of the vesting requirement. After 3 years, if you are still with the company then you have the right to exercise the options. For example, if the stock goes up to $110, you could pay the company $50(1,000) = $50,000 and receive 1,000 shares of stock worth $110,000. However, if you don't exercise the options within 10 years, they will expire and thus be worthless.

Even though the vesting requirement prevents you from exercising the options the moment they are granted to you, the options clearly have some immediate value. Therefore, if you are choosing between different job offers where options are involved, you will need a way to determine the value of the alternative options. This chapter explains how to value options, so read on.

The Intrinsic Value of Stock Options

In previous chapters we showed that the intrinsic value of an asset is the present value of its cash flows. This time value of money approach works great for stocks and bonds, but we must use another approach for op-
tions and derivatives. If we can find a portfolio of stocks and risk-free bonds that replicates an option's cash flows, then the intrinsic value of the option must be identical to the value of the replicating portfolio.

Cost of equity (r_s)

Dividends (D_t)

$$\text{Stock price} = \frac{D_1}{(1 + r_s)^1} + \frac{D_2}{(1 + r_s)^2} + \cdots + \frac{D_\infty}{(1 + r_s)^\infty}$$

Risk-free bond

Portfolio of stock and risk-free bond that replicates cash flows of the option

Value of option must be the same as the replicating portfolio
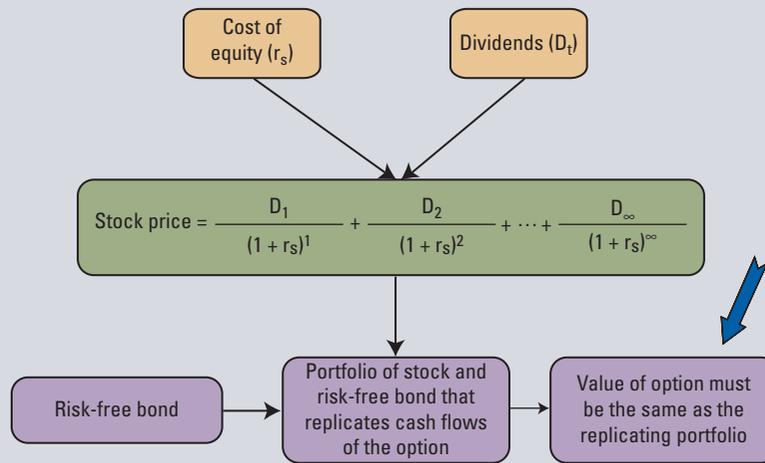
There are two fundamental approaches to valuing assets. The first is the *discounted cash flow* (DCF) approach, which we covered in previous chapters: An asset's value is the present value of its cash flows. The second is the *option pricing* approach. It is important that every manager understands the basic principles of option pricing, for the following reasons. First, many projects allow managers to make strategic or tactical changes in plans as market conditions change. The existence of these "embedded options" often means the difference between a successful project and a failure. Understanding basic financial options can help you manage the value inherent in these real options. Second, many companies use derivatives to manage risk; many derivatives are types of financial options, so an understanding of basic financial options is necessary before tackling derivatives. Third, option pricing theory provides insights into the optimal debt/equity choice, especially when convertible securities are involved. And fourth, understanding financial options will help you better understand any employee stock options that you receive.
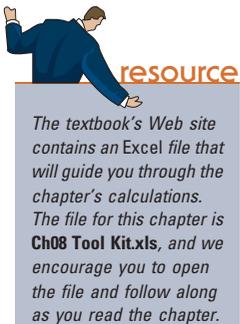
8.1 OVERVIEW OF FINANCIAL OPTIONS

In general, an **option** is a contract that gives its owner the right to buy (or sell) an asset at some predetermined price within a specified period of time. However, there are many types of options and option markets.[1] Consider the options reported in Table 8-1, which is an extract from a Listed Options Quotations table as it might appear on a Web site or in a daily newspaper. The first column reports the closing

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[1]For an in-depth treatment of options, see Don M. Chance, *An Introduction to Derivatives and Risk Management* (Mason, OH: Thomson/South-Western, 2007), or John C. Hull, *Options, Futures, and Other Derivatives*, 7th ed. (Upper Saddle River, NJ: Prentice-Hall, 2009).

TABLE 8-1	Listed Options Quotations for January 8, 2010						
CLOSING PRICE	**STRIKE PRICE**	**CALLS—LAST QUOTE**			**PUTS—LAST QUOTE**		
		FEBRUARY	**MARCH**	**MAY**	**FEBRUARY**	**MARCH**	**MAY**
General Computer Corporation (GCC)							
53.50	50	4.25	4.75	5.50	0.65	1.40	2.20
53.50	55	1.30	2.05	3.15	2.65	r	4.50
53.50	60	0.30	0.70	1.50	6.65	r	8.00

Note: r means not traded on January 8.

stock price. For example, the table shows that General Computer Corporation's (GCC) stock price closed at $53.50 on January 8, 2010.

A **call option** gives its owner the right to *buy* a share of stock at a fixed price, which is called the **strike price** (sometimes called the **exercise price** because it is the price at which you exercise the option). A **put option** gives its owner the right to *sell* a share of stock at a fixed strike price. For example, the first row in Table 8-1 is for GCC's options that have a $50 strike price. Observe that the table has columns for call options and for put options with this strike price.

Each option has an **expiration date**, after which the option may not be exercised. Table 8-1 reports data for options that expire in February, March, and May (the expiration date is the Friday before the third Saturday of the exercise month). If the option can be exercised any time before the expiration, then it is called an **American option**; if it can be exercised only on its expiration date, it is a **European option**. All of GCC's options are American options. The first row shows that GCC has a call option with a strike price of $50 that expires on May 14 (the third Saturday in May 2010 is the 15th). The quoted price for this option is $5.50.[2]

When the current stock price is greater than the strike price, the option is **in-the-money**. For example, GCC's $50 (strike) May call option is in-the-money by $53.50 – $50 = $3.50. Thus, if the option were immediately exercised, it would have a payoff of $3.50. On the other hand, GCC's $55 (strike) May call is **out-of-the-money** because the current $53.50 stock price is below the $55 strike price. Obviously, you currently would not want to exercise this option by paying the $55 strike price for a share of stock selling for $53.50. Therefore, the **exercise value**, which is any profit from immediately exercising an option, is[3]

$$\text{Exercise value} = \text{MAX}[\text{Current price of the stock} - \text{Strike price}, 0] \qquad (8\text{-}1)$$

An option's price always will be greater than (or equal to) its exercise value. If the option's price were less, you could buy the option and immediately exercise it, reaping a sure gain. For example, GCC's May call with a $50 strike price sells for $5.50, which is greater than its exercise value of $3.50. Also, GCC's out-of-the-money May call with a strike price of $55 sells for $3.15 even though it would be worthless if it had to be exercised immediately. An option always will be worth more than zero as

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[2]Option contracts are generally written in 100-share multiples, but we focus on the cost and payoffs of a single option.

[3]MAX means choose the maximum. For example, MAX[15, 0] = 15 and MAX[−10, 0] = 0.

long as there is still any chance at all that it will end up in-the-money: Where there is life, there is hope! The difference between the option's price and its exercise value is called the **time value** because it represents the extra amount over the option's immediate exercise value that a purchaser will pay for the chance the stock price will appreciate over time.[4] For example, GCC's May call with a $50 strike price sells for $5.50 and has an exercise value of $3.50, so its time value is $5.50 − $3.50 = $2.00.

Suppose you bought GCC's $50 (strike) May call option for $5.50 and then the stock price increased to $60. If you exercised the option by purchasing the stock for the $50 strike price, you could immediately sell the share of stock at its market price of $60, resulting in a payoff of $60 − $50 = $10. Notice that the stock itself had a return of 12.1% = ($60 − $53.50)/$53.50, but the option's return was 81.8% = ($10 − $5.50)/$5.50. Thus, the option offers the possibility of a higher return.

However, if the stock price fell to $50 and stayed there until the option expired, the stock would have a return of −6.5% = ($50.00 − $53.50)/$53.50, but the option would have a 100% loss (it would expire worthless). As this example shows, call options are a lot riskier than stocks. This works to your advantage if the stock price goes up but to your disadvantage if the stock price falls.

Suppose you bought GCC's May put option (with a strike price of $50) for $2.20 and then the stock price fell to $45. You could buy a share of stock for $45 and exercise the put option, which would allow you to sell the share of stock at its strike price of $50. Your payoff from exercising the put would be $5 = $50 − $45. Stockholders would lose money because the stock price fell, but a put holder would make money. In this example, your rate of return would be 127.3% = ($5 − $2.20)/$2.20. So if you think a stock price is going to fall, you can make money by purchasing a put option. On the other hand, if the stock price doesn't fall below the strike price of $50 before the put expires, you would lose 100% of your investment in the put option.[5]

Options are traded on a number of exchanges, with the Chicago Board Options Exchange (CBOE) being the oldest and the largest. Existing options can be traded in the secondary market in much the same way that existing shares of stock are traded in secondary markets. But unlike new shares of stock that are issued by corporations, new options can be "issued" by investors. This is called **writing** an option.

For example, you could write a call option and sell it to some other investor. You would receive cash from the option buyer at the time you wrote the option, but you would be obligated to sell a share of stock at the strike price if the option buyer later decided to exercise the option.[6] Thus, each option has two parties, the writer and the buyer, with the CBOE (or some other exchange) acting as an intermediary. Other than commissions, the writer's profits are exactly opposite those of the buyer. An investor who writes call options against stock held in his or her portfolio is said to be selling **covered options**. Options sold without the stock to back them up are called **naked options**.

In addition to options on individual stocks, options are also available on several stock indexes such as the NYSE Index and the S&P 100 Index. Index options permit one to hedge (or bet) on a rise or fall in the general market as well as on individual stocks.

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[4]Among traders, an option's market price is also called its "premium." This is particularly confusing since for all other securities the word *premium* means the excess of the market price over some base price. To avoid confusion, we will not use the word *premium* to refer to the option price.

[5]Most investors don't actually exercise an option prior to expiration. If they want to cash in the option's profit or cut its losses, they sell the option to some other investor. As you will see later in the chapter, the cash flow from selling the option before its expiration is always greater than (or equal to) the profit from exercising the option.

[6]Your broker would require collateral to ensure that you kept this obligation.

Financial Reporting for Employee Stock Options

When granted to executives and other employees, options are a "hybrid" form of compensation. At some companies, especially small ones, option grants may be a substitute for cash wages: employees are willing to take lower cash salaries if they have options. Options also provide an incentive for employees to work harder. Whether issued to motivate employees or to conserve cash, options clearly have value at the time they are granted, and they transfer wealth from existing shareholders to employees to the extent that they do not reduce cash expenditures or increase employee productivity enough to offset their value at the time of issue.

Companies like the fact that an option grant requires no immediate cash expenditure, although it might dilute shareholder wealth if later it is exercised. Employees, and especially CEOs, like the potential wealth that they receive when they are granted options. When option grants were relatively small, they didn't show up on investors' radar screens. However, as the high-tech sector began making mega-grants in the 1990s, and as other industries followed suit in the heavy use of options, stockholders began to realize that large grants were making some CEOs filthy rich at the stockholders' expense.

Before 2005, option grants were barely visible in companies' financial reports. Even though such grants are clearly a wealth transfer to employees, companies were required only to footnote the grants and could ignore them when reporting their income statements and balance sheets. The Financial Accounting Standards Board now requires companies to show option grants as an expense on the income statement. To do this, the value of the options is estimated at the time of the grant and then expensed during the vesting period, which is the amount of time the employee must wait before being allowed to exercise the options. For example, if the initial value is $100 million and the vesting period is 2 years, the company would report a $50 million expense for each of the next 2 years. This approach isn't perfect, because the grant is not a cash expense; nor does the approach take into account changes in the option's value after the initial grant. However, it does make the option grant more visible to investors, which is a good thing.

The leverage involved in option trading makes it possible for speculators with just a few dollars to make a fortune almost overnight. Also, investors with sizable portfolios can sell options against their stocks and earn the value of the option (less brokerage commissions) even if the stock's price remains constant. Most important, though, options can be used to create *hedges* that protect the value of an individual stock or portfolio.[7]

Conventional options are generally written for 6 months or less, but a type of option called a **Long-Term Equity AnticiPation Security (LEAPS)** is different. Like conventional options, LEAPS are listed on exchanges and are available on both individual stocks and stock indexes. The major difference is that LEAPS are long-term options, having maturities of up to almost 3 years. One-year LEAPS cost about twice as much as the matching 3-month option, but because of their much longer time to expiration, LEAPS provide buyers with more potential for gains and offer better long-term protection for a portfolio.

Corporations on whose stocks the options are written have nothing to do with the option market. Corporations do not raise money in the option market, nor do they have any direct transactions in it. Moreover, option holders do not vote for corporate directors or receive dividends. There have been studies by the SEC and others as to

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[7]Insiders who trade illegally generally buy options rather than stock because the leverage inherent in options increases the profit potential. However, it is illegal to use insider information for personal gain, and an insider using such information would be taking advantage of the option seller. Insider trading, in addition to being unfair and essentially equivalent to stealing, hurts the economy: Investors lose confidence in the capital markets and raise their required returns because of an increased element of risk, and this raises the cost of capital and thus reduces the level of real investment.

whether option trading stabilizes or destabilizes the stock market and whether this activity helps or hinders corporations seeking to raise new capital. The studies have not been conclusive, but research on the impact of option trading is ongoing.

What is an option? A call option? A put option?

Define a call option's exercise value. Why is the market price of a call option usually above its exercise value?

Brighton Memory's stock is currently trading at $50 a share. A call option on the stock with a $35 strike price currently sells for $21. What is the exercise value of the call option? **($15.00)** What is the time value? **($6.00)**

8.2 THE SINGLE-PERIOD BINOMIAL OPTION PRICING APPROACH

We can use a model like the Capital Asset Pricing Model (CAPM) to calculate the required return on a stock and then use that required return to discount its expected future cash flows to find its value. No such model exists for the required return on options, so we must use a different approach to find an option's value. In Section 8.5 we describe the Black-Scholes option pricing model, but in this section we explain the binomial option pricing model. The idea behind this model is different from that of the DCF model used for stock valuation. Instead of discounting cash flows at a required return to obtain a price, as we did with the stock valuation model, we will use the option, shares of stock, and the risk-free rate to construct a portfolio whose value we already know and then deduce the option's price from this portfolio's value.

The following sections describe and apply the binomial option pricing model to Western Cellular, a manufacturer of cell phones. Call options exist that permit the holder to buy 1 share of Western at a strike price, X, of $35. Western's options will expire at the end of 6 months (t is the number of years until expiration, so t = 0.5 for Western's options).Western's stock price, P, is currently $40 per share. Given this background information, we will use the binomial model to determine the call option's value. The first step is to determine the option's possible payoffs, as described in the next section.

Payoffs in a Single-Period Binomial Model

In general, the time until expiration can be divided into many periods, with n denoting the number of periods. But in a single-period model, which we describe in this section, there is only one period. We assume that, at the end of the period, the stock's price can take on only one of two possible values, so this is called the **binomial approach**. For this example, Western's stock will either go up (u) by a factor of 1.25 or go down (d) by a factor of 0.80. If we were considering a riskier stock, then we would have assumed a wider range of ending prices; we will show how to estimate this range later in the chapter. If we let u = 1.25 and d = 0.80, then the ending stock price will be either P(u) = $40(1.25) = $50 or P(d) = $40(0.80) = $32. Figure 8-1 illustrates the stock's possible price paths and contains additional information about the call option that is explained in what follows.

When the option expires at the end of the year, Western's stock will sell for either $50 or $32. As shown in Figure 8-1, if the stock goes up to $50 then the option will have a payoff, C_u, of $15 at expiration because the option is in-the-money: $50 – $35 = $15. If the stock price goes down to $32 then the option's payoff, C_d, will be zero because the option is out-of-the-money.

FIGURE 8-1   Binomial Payoffs from Holding Western Cellular's Stock or Call Option

The Hedge Portfolio Approach

Suppose we created a portfolio by writing 1 call option and purchasing 1 share of stock. As Figure 8-1 shows, if the stock price goes up then our portfolio's stock will be worth $50 but we will owe $15 on the option, so our portfolio's net payoff is $35 = $50 − $15. If the stock price goes down then our portfolio's stock will be worth only $32, but the amount we owe on the written option also will fall to zero, leaving the portfolio's net payoff at $32. The portfolio's end-of-period price range is smaller than if we had just owned the stock, so writing the call option reduces the portfolio's price risk. Taking this further: Is it possible for us to choose the number of shares held by our portfolio so that it will have the same net payoff whether the stock goes up or down? If so, then our portfolio is hedged and will have a riskless payoff when the option expires. Therefore, it is called a **hedge portfolio**.

   We are not really interested in investing in the hedge portfolio, but we want to use it to help us determine the value of the option. Notice that if the hedge portfolio has a riskless net payoff when the option expires, then we can find the present value of this payoff by discounting it at the risk-free rate. Our current portfolio value must equal this present value, which allows us to determine the option's value. The following example illustrates the steps in this approach.

1. Find N_s, the number of shares of stock in the hedge portfolio.   We want the portfolio's payoff to be the same whether the stock goes up or down. If we write 1 call option and buy N_s shares of stock, then the portfolio's stock will be worth $N_s(P)(u)$ should the stock price go up, so its net payoff will be $N_s(P)(u) − C_u$. The portfolio's stock will be worth $N_s(P)(d)$ if the stock price goes down, so its net payoff will be

$N_s(P)(d) - C_d$. Setting these portfolio payoffs equal to one another and then solving for N_s yields

$$N_s = \frac{C_u - C_d}{P(u) - P(d)} = \frac{C_u - C_d}{P(u - d)} \tag{8-2}$$

For Western, the hedge portfolio has 0.83333 share of stock:[8]

$$N_s = \frac{C_u - C_d}{P(u) - P(d)} = \frac{\$15 - \$0}{\$50 - \$32} = 0.83333$$

2. Find the hedge portfolio's payoff.   Our next step is to find the hedge portfolio's payoff when the stock price goes up (you will get the same result if instead you find the portfolio's payoff when the stock goes down). Recall that the hedge portfolio has N_s shares of stock and that we have written the call option, so the call option's payoff must be subtracted:

$$\text{Hedge portfolio's payoff if stock is up} = N_s P(u) - C_u$$
$$= 0.83333(\$50) - \$15$$
$$= \$26.6665$$

$$\text{Hedge portfolio's payoff if stock is down} = N_s P(d) - C_d$$
$$= 0.83333(\$32) - \$0$$
$$= \$26.6665$$

Figure 8-2 illustrates the payoffs of the hedge portfolio.

3. Find the present value of the hedge portfolio's payoff.   Because the hedge portfolio's payoff is riskless, the current value of the hedge portfolio must be equal to the present value of its riskless payoff. Option pricing models usually assume continuous compounding, which we discuss in **Web Extension 4C** on the textbook's Web site, but daily compounding works well. For a 1-period model, the time to expiration also is the time until the payoff occurs. In a later section we consider more than 1 period prior to expiration, so the time that we discount the payoff is equal to the time until expiration (t) divided by the number of periods until expiration (n). In our example, t = 0.5 and n = 1. Therefore, the present value of the hedge portfolio's payoff is

$$\begin{matrix}\text{PV of riskless} \\ \text{payoff}\end{matrix} = \frac{\$26.6665}{\left(1 + \frac{r_{RF}}{365}\right)^{365(t/n)}} = \frac{\$26.6665}{\left(1 + \frac{0.08}{365}\right)^{365(0.5/1)}}$$
$$= \$25.621$$

4. Find the option's value.   The current value of the hedge portfolio is the value of the stock, $N_s(P)$, less the value of the call option we wrote. Because the payoff is riskless, the current value of the hedge portfolio must also equal the present value of the riskless payoff:

$$\begin{matrix}\text{Current value of} \\ \text{hedge portfolio}\end{matrix} = N_s P - V_c = \begin{matrix}\text{Present value of} \\ \text{riskless payoff}\end{matrix}$$

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[8]An easy way to remember this formula is to notice that N_s is equal to the range in possible option payoffs divided by the range in possible stock prices.

FIGURE 8-2   Hedge Portfolio with Riskless Payoffs

	A	B	C	D	E	F
180					Strike price: X =	$35.00
181					Current stock price: P =	$40.00
182					Up factor for stock price: u =	1.25
183					Down factor for stock price: d =	0.80
184					Up option payoff: C_u = MAX[0,P(u)–X] =	$15.00
185					Down option payoff: C_d =MAX[0,P(d)–X] =	$0.00
186				Number of shares of stock in portfolio: N_s = (C_u - C_d) / P(u–d) =		0.83333
187						
188				Stock price = P (u) = $50.00		
189				Portfolio's stock payoff: = P(u)(N_s) =		$41.67
190	P,			Subtract option's payoff: C_u =		$15.00
191	current			Portfolio's net payoff = P(u)N_s – C_u =		$26.67
192	stock price					
193	$40					
194						
195						
196				Stock price = P (d) = $32.00		
197				Portfolio's stock payoff: = P(d)(N_s) =		$26.67
198				Subtract option's payoff: C_d =		$0.00
199				Portfolio's net payoff = P(d)N_s – C_d =		$26.67

Solving for the call option's value, we get

$$V_C = N_s P - \frac{\text{Present value of}}{\text{riskless payoff}}$$

For Western's option, this is

$$V_C = 0.83333(\$40) - \$25.621$$
$$= \$7.71$$

Hedge Portfolios and Replicating Portfolios

In our previous derivation of the call option's value, we combined an investment in the stock with writing a call option to create a risk-free investment. We can modify this approach and create a portfolio that replicates the call option's payoffs. For example, suppose we formed a portfolio by purchasing 0.83333 shares of Western's stock and borrowing $25.621 at the risk-free rate (this is equivalent to selling a T-bill short). In 6 months we would repay $25.621(1 + 0.08/365)^{365(0.5/1)}$ = $26.6665. If the stock goes up, our net payoff would be 0.83333($50) – $26.6665 = $15.00. If the stock goes down, our net payoff would be 0.83333($32) – $26.6665 = $0. The portfolio's payoffs are exactly equal to the option's payoffs as shown in Figure 8-1, so our portfolio of 0.83333 shares of stock and the $25.621 that we borrowed would exactly replicate the option's payoffs. Therefore, this is called a **replicating portfolio**. Our cost to create this portfolio is the cost of the stock less the amount we borrowed:

Cost of replicating portfolio = 0.83333($40) – $25.621 = $7.71

If the call option did not sell for exactly $7.71, then a clever investor could make a sure profit. For example, suppose the option sold for $8. The investor would write an option, which would provide $8 of cash now but would obligate the investor to pay either $15 or $0 in 6 months when the option expires. However, the investor could use the $8 to create the replicating portfolio, leaving the investor with $8 − $7.71 = $0.29. In 6 months, the replicating portfolio will pay either $15 or $0. Thus, the investor isn't exposed to any risk—the payoffs received from the replicating portfolio exactly offset the payoffs owed on the option. The investor uses none of his own money, has no risk, has no net future obligations, but has $0.29 in cash. This is **arbitrage**, and if such an arbitrage opportunity existed then the investor would scale it up by writing thousands of options.[9]

Such arbitrage opportunities don't persist for long in a reasonably efficient economy because other investors will also see the opportunity and will try to do the same thing. With so many investors trying to write (i.e., sell) the option, its price will fall; with so many investors trying to purchase the stock, its price will increase. This will continue until the option and replicating portfolio have identical prices. And because our financial markets are really quite efficient, you would never observe the derivative security and the replicating portfolio trading for different prices—they would always have the same price and there would be no arbitrage opportunities. What this means is that, by finding the price of a portfolio that replicates a derivative security, we have also found the price of the derivative security itself!

Self-Test	Describe how a risk-free hedge portfolio can be created using stocks and options. How can such a portfolio be used to help estimate a call option's value?

Describe how a risk-free hedge portfolio can be created using stocks and options. How can such a portfolio be used to help estimate a call option's value?

What is a replicating portfolio, and how is it used to find the value of a derivative security?

What is arbitrage?

Lett Incorporated's stock price is now $50, but it is expected either to rise by a factor of 1.5 or fall by a factor of 0.7 by the end of the year. There is a call option on Lett's stock with a strike price of $55 and an expiration date 1 year from now. What are the stock's possible prices at the end of the year? **($75 or $35)** What is the call option's payoff if the stock price goes up? **($20)** If the stock price goes down? **($0)** If we sell one call option, how many shares of Lett's stock must we buy to create a riskless hedged portfolio consisting of the option position and the stock? **(0.5)** What is the payoff of this portfolio? **($17.50)** If the annual risk free rate is 6%, then how much is the riskless portfolio worth today (assuming daily compounding)? **($16.48)** What is the current value of the call option? **($8.52)**

8.3 THE SINGLE-PERIOD BINOMIAL OPTION PRICING FORMULA[10]

The hedge portfolio approach works well if you only want to find the value of one type of option with one period until expiration. But in all other situations, the step-by-step approach becomes tedious very quickly. The following sections describe a formula that replaces the step-by-step approach.

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[9]If the option sold for less than the replicating portfolio, the investor would raise cash by shorting the portfolio and use the cash to purchase the option, again resulting in arbitrage profits.

[10]The material in this section is relatively technical, and some instructors may choose to skip it with no loss in continuity.

The Binomial Option Pricing Formula

With a little (or a lot!) of algebra, we can derive a single formula for a call option. After programming it into *Excel*, which we did for this chapter's ***Tool Kit***, it is easy to change inputs and determine the new value of a call option. Here is the binomial option pricing formula:

$$V_C = \frac{C_u\left[\dfrac{(1 + r_{RF}/365)^{365(t/n)} - d}{u - d}\right] + C_d\left[\dfrac{u - (1 + r_{RF}/365)^{365(t/n)}}{u - d}\right]}{(1 + r_{RF}/365)^{365(t/n)}} \tag{8-3}$$

We can apply this formula to Western's call option:

$$V_C = \frac{\$15\left[\dfrac{(1 + 0.08/365)^{365(0.5/1)} - 0.80}{1.25 - 0.80}\right] + \$0\left[\dfrac{1.25 - (1 + 0.08/365)^{365(0.5/1)}}{1.25 - 0.80}\right]}{(1 + 0.08/365)^{365(0.5/1)}}$$

$$= \frac{\$15(0.5351) + \$0(0.2092)}{1.040806} = \$7.71$$

Notice that this is the same value that resulted from the step-by-step process shown earlier.

The binomial option pricing formula in Equation 8-3 does not include the actual probabilities that the stock will go up or down, nor does it include the expected stock return, which is not what one might expect. After all, the higher the stock's expected return, the greater the chance that the call will be in-the-money at expiration. Note, however, that the stock's expected return is already indirectly incorporated into the stock price.

If we want to value other Western call options or puts that expire in 6 months, then we can again use Equation 8-3. Observe that for options with the same time left until expiration, C_u and C_d are the only variables that depend on the option itself. The other variables depend only on the stock process (u and d), the risk-free rate, the time until expiration, and the number of periods until expiration. If we group these variables together, we can then define π_u and π_d as

$$\pi_u = \frac{\left[\dfrac{(1 + r_{RF}/365)^{365(t/n)} - d}{u - d}\right]}{(1 + r_{RF}/365)^{365(t/n)}} \tag{8-4}$$

and

$$\pi_d = \frac{\left[\dfrac{u - (1 + r_{RF}/365)^{365(t/n)}}{u - d}\right]}{(1 + r_{RF}/365)^{365(t/n)}} \tag{8-5}$$

By substituting these values into Equation 8-3, we obtain an option pricing model that can be applied to all of Western's 6-month options:

$$V_C = C_u\pi_u + C_d\pi_d \tag{8-6}$$

In this example, π_u and π_d are

$$\pi_u = \frac{\left[\dfrac{(1 + 0.08/365)^{365(0.5/1)} - 0.80}{1.25 - 0.80}\right]}{(1 + 0.08/365)^{365(0.5/1)}} = 0.5141$$

and

$$\pi_d = \frac{\left[\dfrac{1.25 - (1 + 0.08/365)^{365(0.5/1)}}{1.25 - 0.80}\right]}{(1 + 0.08/365)^{365(0.5/1)}} = 0.4466$$

Using Equation 8-6, the value of Western's 6-month call option with a strike price of $35 is

$$V_c = C_u\pi_u + C_d\pi_d$$
$$= \$15(0.5141) + \$0(0.4466)$$
$$= \$7.71$$

Sometimes these π's are called *primitive securities* because π_u is the price of a simple security that pays $1 if the stock goes up and nothing if it goes down; π_u is the opposite. This means that we can use these π's to find the price of any 6-month option on Western. For example, suppose we want to find the value of a 6-month call option on Western but with a strike price of $30. Rather than reinvent the wheel, all we have to do is find the payoffs of this option and use the same values of π_u and π_d in Equation 8-6. If the stock goes up to $50, the option will pay $50 − $30 = $20; if the stock falls to $32, the option will pay $32 − $30 = $2. The value of the call option is:

$$\text{Value of 6-month call with \$30 strike price} = C_u\pi_u + C_d\pi_d$$
$$= \$20(0.5141) + \$2(0.4466)$$
$$= \$11.18$$

It is a bit tedious initially to calculate π_u and π_d, but once you save them it is easy to find the value of any 6-month call or put option on the stock. In fact, you can use these π's to find the value of any security with payoffs that depend on Western's 6-month stock prices, which makes them a very powerful tool.

Self-Test

Yegi's Fine Phones has a current stock price of $30. You need to find the value of a call option with a strike price of $32 that expires in 3 months. Use the binomial model with one period until expiration. The factor for an increase in stock price is u = 1.15; the factor for a downward movement is d = 0.85. What are the possible stock prices at expiration? **($34.50 or $25.50)** What are the option's possible payoffs at expiration? **($2.50 or $0)** What are π_u and π_d? **(0.5422 and 0.4429)** What is the current value of the option (assume each month is 1/12 of a year)? **($1.36)**

8.4 THE MULTI-PERIOD BINOMIAL OPTION PRICING MODEL[11]

Clearly, this example is simplified. Although you could duplicate buying 0.8333 share and writing one option by buying 8,333 shares and writing 10,000 options, the stock price assumptions are unrealistic—Western's stock price could be almost anything

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[11]The material in this section is relatively technical, and some instructors may choose to skip it with no loss in continuity.

after 6 months, not just $50 or $32. However, if we allowed the stock to move up or down more often, then a more realistic range of ending prices would result. In other words, dividing the time until expiration into more periods would improve the realism of the resulting prices at expiration. The key to implementing a multi-period binomial model is to keep the stock return's annual standard deviation the same no matter how many periods you have during a year. In fact, analysts typically begin with an estimate of the standard deviation and use it to determine u and d. The derivation is beyond the scope of a financial management textbook, but the appropriate equations are

$$u = e^{\sigma\sqrt{t/n}} \tag{8-7}$$

$$d = \frac{1}{u} \tag{8-8}$$

where σ is the annualized standard deviation of the stock's return, t is the time in years until expiration, and n is the number of periods until expiration.

The standard deviation of Western's stock returns is 31.5573%, and application of Equations 8-7 and 8-8 confirms the values of u and d that we used previously:

$$u = e^{0.315573\sqrt{0.5/1}} = 1.25 \quad \text{and} \quad d = \frac{1}{1.25} = 0.80$$

Now suppose we allow stock prices to change every 3 months (which is 0.25 years). Using Equations 8-7 and 8-8, we estimate u and d to be

$$u = e^{0.315573\sqrt{0.5/2}} = 1.1709 \quad \text{and} \quad d = \frac{1}{1.1709} = 0.8540$$

At the end of the first 3 months, Western's price would either rise to $40(1.1709) = $46.84 or fall to $40(0.8540) = $34.16. If the price rises in the first 3 months to $46.84, then it would either go up to $46.84(1.1709) = $54.84 or go down to $46.84(0.8540) = $40 at expiration. If instead the price initially falls to $40(0.8540) = $34.16 during the first 3 months, then it would either go up to $34.16(1.1709) = $40 or go down to $34.16(0.8540) = $29.17 by expiration. This pattern of stock price movements is called a **binomial lattice** and is shown in Figure 8-3.

Because the interest rate and the volatility (as defined by u and d) are constant for each period, we can calculate π_u and π_d for any period and apply these same values for each period:[12]

$$\pi_u = \frac{\left[\dfrac{(1 + 0.08/365)^{365(0.5/2)} - 0.8540}{1.1709 - 0.8540}\right]}{(1 + 0.08/365)^{365(0.5/2)}} = 0.51400$$

$$\pi_d = \frac{\left[\dfrac{1.1709 - (1 + 0.08/365)^{365(0.5/2)}}{1.1709 - 0.8540}\right]}{(1 + 0.08/365)^{365(0.5/2)}} = 0.46621$$

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[12]These values were calculated in *Excel*, so there may be small differences due to rounding in intermediate steps.

FIGURE 8-3 Two-Period Binomial Lattice and Option Valuation

	A	B	C	D	E	F	G	H	I
435		Standard deviation of stock return: σ =			31.557%				
436		Current stock price: P =			$40.00				
437		Up factor for stock price: u =			1.1709				
438		Down factor for stock price: d =			0.8540				
439		Strike price: X =			$35.00				
440		Risk-free rate: r_{RF} =			8.00%				
441		Years to expiration: t =			0.50				
442		Number of periods until expiration: n =			2				
443		Price of $1 payoff if stock goes up: π_u =			0.51400				
444		Price of $1 payoff if stock goes down: π_d =			0.46621				
445									
446		Now			3 months			6 months	

Binomial lattice:

Now: P = $40.00; $V_C = C_u\pi_u + C_d\pi = \$7.64$

3 months (up): Stock = P (u) = $46.84; $C_u = C_{uu}\pi_u + C_{ud}\pi_d = \$12.53$

3 months (down): Stock = P (d) = $34.16; $C_d = C_{ud}\pi_u + C_{dd}\pi_d = \$2.57$

6 months (uu): Stock = P (u) (u) = $54.84; $C_{uu} = \text{Max}[P(u)(u) - X, 0] = \$19.84$

6 months (ud): Stock = P (u) (d) = P (d) (u) = $40.00; $C_{ud} = C_{du} = \text{Max}[P(u)(d) - X, 0] = \$5.00$

6 months (dd): Stock = P (d) (d) = $29.17; $C_{dd} = \text{Max}[P(d)(d) - X, 0] = \$0.00$

These values are shown in Figure 8-3.

The lattice shows the possible stock prices at the option's expiration and we know the strike price, so we can calculate the option payoffs at expiration. Figure 8-3 also shows the option payoffs at expiration. If we focus only on the upper right portion of the lattice shown inside the dotted lines, then it is similar to the single-period problem we solved in Section 8.3. In fact, we can use the binomial option pricing model from Equation 8-6 to determine the value of the option in 3 months given that the stock price increased to $46.84. As shown in Figure 8-3, the option will be worth $12.53 in 3 months if the stock price goes up to $46.84. We can repeat this procedure on the lower right portion of Figure 8-3 to determine the call option's value in 3 months if the stock price falls to $34.16; in this case, the call's value would be $2.57. Finally, we can use Equation 8-6 and the 3-month option values just calculated to determine the current price of the option, which is $7.64. Thus, we are able to find the current option price by solving three simple binomial problems.

If we broke the year into smaller periods and allowed the stock price to move up or down more often, then the lattice would have an even more realistic range of possible ending stock prices. Of course, estimating the current option price would require solving lots of binomial problems within the lattice, but each problem is simple and computers can solve them rapidly. With more outcomes, the resulting

estimated option price is more accurate. For example, if we divide the year into 15 periods then the estimated price is $7.42. With 50 periods, the price is $7.39. With 100 periods it is still $7.39, which shows that the solution converges to its final value within a relatively small number of steps. In fact, as we break the time to expiration into smaller and smaller periods, the solution for the binomial approach converges to the Black-Scholes solution, which is described in the next section.

The binomial approach is widely used to value options with more complicated payoffs than the call option in our example, such as employee stock options. This is beyond the scope of a financial management textbook, but if you are interested in learning more about the binomial approach then you should take a look at the textbooks by Don Chance and John Hull cited in footnote 1.

Self-Test

Ringling Cycle's stock price is now $20. You need to find the value of a call option with a strike price of $22 that expires in 2 months. You want to use the binomial model with 2 periods (each period is a month). Your assistant has calculated that u = 1.1553, d = 0.8656, π_u = 0.4838, and π_d = 0.5095. Draw the binomial lattice for stock prices. What are the possible prices after 1 month? **($23.11 or $17.31)** After 2 months? **($26.69, $20, or $14.99)** What are the option's possible payoffs at expiration? **($4.69, $0, or $0)** What will the option's value be in 1 month if the stock goes up? **($2.27)** What will the option's value be in 1 month if the stock price goes down? **($0)** What is the current value of the option (assume each month is 1/12 of a year)? **($1.10)**

8.5 The Black-Scholes Option Pricing Model (OPM)

The **Black-Scholes option pricing model (OPM)**, developed in 1973, helped give rise to the rapid growth in options trading. This model, which has even been programmed into some handheld and Web-based calculators, is widely used by option traders.

OPM Assumptions and Equations

In deriving their option pricing model, Fischer Black and Myron Scholes made the following assumptions.

1. The stock underlying the call option provides no dividends or other distributions during the life of the option.
2. There are no transaction costs for buying or selling either the stock or the option.
3. The short-term, risk-free interest rate is known and is constant during the life of the option.
4. Any purchaser of a security may borrow any fraction of the purchase price at the short-term, risk-free interest rate.
5. Short selling is permitted, and the short seller will receive immediately the full cash proceeds of today's price for a security sold short.
6. The call option can be exercised only on its expiration date.
7. Trading in all securities takes place continuously, and the stock price moves randomly.

The derivation of the Black-Scholes model rests on the same concepts as the binomial model, except time is divided into such small increments that stock prices

change continuously. The Black-Scholes model consists of the following three equations:

$$V_C = P[N(d_1)] - Xe^{-r_{RF}t}[N(d_2)] \qquad (8\text{-}9)$$

$$d_1 = \frac{\ln(P/X) + [r_{RF} + (\sigma^2/2)]t}{\sigma\sqrt{t}} \qquad (8\text{-}10)$$

$$d_2 = d_1 - \sigma\sqrt{t} \qquad (8\text{-}11)$$

The variables used in the Black-Scholes model are explained below.

WWW

Robert's Online Option Pricer can be accessed at **http://www.intrepid.com/ robertl/index.html**. *The site is designed to provide a financial service over the Internet to small investors for option pricing, giving anyone a means to price option trades without having to buy expensive software and hardware.*

V_C = Current value of the call option.

P = Current price of the underlying stock.

$N(d_i)$ = Probability that a deviation less than d_i will occur in a standard normal distribution. Thus, $N(d_1)$ and $N(d_2)$ represent areas under a standard normal distribution function.

X = Strike price of the option.

$e \approx 2.7183$.

r_{RF} = Risk-free interest rate.[13]

t = Time until the option expires (the option period).

$\ln(P/X)$ = Natural logarithm of P/X.

σ = Standard deviation of the rate of return on the stock.

The value of the option is a function of five variables: (1) P, the stock's price; (2) t, the option's time to expiration; (3) X, the strike price; (4) σ, the standard deviation of the underlying stock; and (5) r_{RF}, the risk-free rate. We do not derive the Black-Scholes model—the derivation involves some extremely complicated mathematics that go far beyond the scope of this text. However, it is not difficult to use the model. Under the assumptions set forth previously, if the option price is different from the one found by Equation 8-9, then this would provide the opportunity for arbitrage profits, which would force the option price back to the value indicated by the model.[14] As we noted earlier, the Black-Scholes model is widely used by traders because actual option prices conform reasonably well to values derived from the model.

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[13]The risk-free rate should be expressed as a continuously compounded rate. If r is a continuously compounded rate, then the effective annual yield is $e^r - 1.0$. An 8% continuously compounded rate of return yields $e^{0.08} - 1 = 8.33\%$. In all of the Black-Scholes option pricing model examples, we will assume that the rate is expressed as a continuously compounded rate.

[14]*Programmed trading*, in which stocks are bought and options are sold (or vice versa), is an example of arbitrage between stocks and options.

Application of the Black-Scholes Option Pricing Model

The current stock price (P), the exercise price (X), and the time to maturity (t) can all be obtained from a newspaper, such as *The Wall Street Journal*, or from the Internet, such as the CBOE's Web site. The risk-free rate (r_{RF}) is the yield on a Treasury bill with a maturity equal to the option expiration date. The annualized standard deviation of stock returns (σ) can be estimated from daily stock prices. First, find the stock return for each trading day for a sample period, such as each trading day of the past year. Second, estimate the variance of the daily stock returns. Third, multiply this estimated daily variance by the number of trading days in a year, which is approximately 250.[15] Take the square root of the annualized variance, and the result is an estimate of the annualized standard deviation.

We will use the Black-Scholes model to estimate Western's call option that we discussed previously. Here are the inputs:

$$P = \$40$$
$$X = \$35$$
$$t = 6 \text{ months } (0.5 \text{ years})$$
$$r_{RF} = 8.0\% = 0.080$$
$$\sigma = 31.557\% = 0.31557$$

Given this information, we first estimate d_1 and d_2 from Equations 8-10 and 8-11:

$$d_1 = \frac{\ln(\$40/\$35) + [0.08 + ((0.31557^2)/2)](0.5)}{0.31557\sqrt{0.5}}$$

$$= \frac{0.13353 + 0.064896}{0.22314} = 0.8892$$

$$d_2 = d_1 - 0.31557\sqrt{0.5} = 0.6661$$

Note that $N(d_1)$ and $N(d_2)$ represent areas under a standard normal distribution function. The easiest way to calculate this value is with *Excel*. For example, we can use the function =**NORMSDIST(0.8892)**, which returns a value of $N(d_1) =$

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[15]If stocks traded every single day of the year, then each daily stock return would cover a period of 24 hours. Suppose you take a sample of these 24-hour returns and estimate the variance. There are 365 24-hour periods in a year, so you should multiply the 24-hour variance by 365 to estimate the annual variance. However, stocks don't trade every day because of weekends and holidays. If you excluded weekends from your sample (i.e., if discarded the returns from the close of trading on Friday to the close of trading on Monday), then each return would be for a 24-hour period. So you should multiply your estimated 24-hour variance by 365 to estimate the annual variance.

If instead you measure returns from the close of one trading day until the close of the next trading day, then some returns are for 24 hours (such as Thursday close to Friday close) and some are for longer periods, like the 72-hour return from Friday close to Monday close. On average, though, five sequential returns cover one week. With roughly 50 weeks of trading in the year (assuming that about 14 weekdays have no trading because of holidays), each of the returns measured from trading day to trading day covers about $1/250 = 1/(5 \times 50)$ year. So if you include returns over weekends and holidays in your sample, you should multiply the variance of daily (i.e., trading-day-to-trading-day) returns by 250 to convert it to an annual variance.

You could use trading-day-to-trading-day returns and adjust them for the length of each period (24 hours, 48, hours, etc.), but most analysts just multiply the variance by 250. Also, some analysts estimate the daily return as $\ln(P_t/P_{t-1})$ instead of estimating the daily return as the percentage change in stock prices.

FIGURE 8-4   Western Cellular's Call Options with a Strike Price of $35

See **Ch08 Tool Kit.xls**.
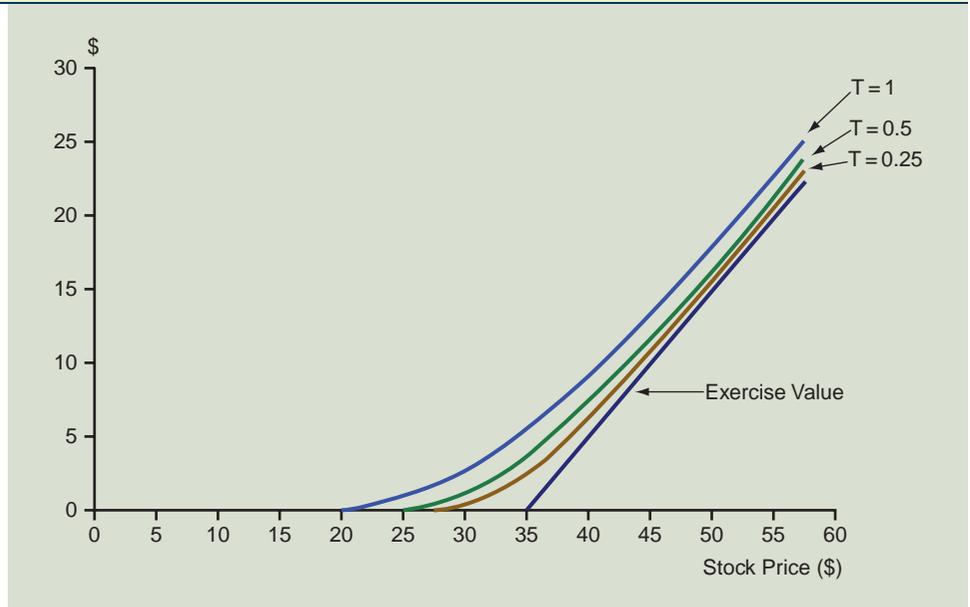
N(0.8892) = 0.8131. Similarly, the **NORMSDIST** function returns a value of $N(d_2)$ = 0.7473.[16] We can use those values to solve Equation 8-9:

$$V_C = \$40[N(0.8892)] - \$35e^{-(0.08)(0.5)}[N(0.6661)]$$

$$= \$7.39$$

Thus, the value of the option is $7.39. This is the same value we found using the binomial approach with 100 periods in the year.

The Five Factors That Affect Option Prices

The Black-Scholes model has five inputs, so there are five factors that affect option prices. Figure 8-4 shows how three of Western Cellular's call options are affected by Western's stock price (all three options have a strike price of $35). The three options expire in 1 year, in 6 months (0.5 years, like the option in our example), and in 3 months (or 0.25 years), respectively.

Figure 8-4 offers several insights regarding option valuation. Notice that for all stock prices, the option prices are always above the exercise value. If this were not true, then an investor could purchase the option and immediately exercise it for a quick profit.

When the stock price falls far below the strike price, the option prices fall toward zero. In other words, options lose value as they become more and more out-of-the-money. When the stock price greatly exceeds the strike price, the option prices fall toward the exercise value. Thus, for very high stock prices, options tend to move up and down by about the same amount as does the stock price.

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[16]If you do not have access to *Excel*, then you can use the table in Appendix D. For example, the table shows that the value for d = 0.88 is 0.5000 + 0.3106 = 0.8106 and that the value for d = 0.89 is 0.5000 + 0.3133 = 0.8133, so N(0.8892) lies between 0.8106 and 0.8133. You could interpolate to find a closer value, but we suggest using *Excel* instead.

Option prices increase if the stock price increases. This is because the strike price is fixed, so an increase in stock price increases the chance that the option will be in-the-money at expiration. Although we don't show it in the figure, an increase in the strike price would obviously cause a decrease in the option's value because higher strike prices mean a lower chance of being in-the-money at expiration.

The 1-year option always has a greater value than the 6-month option, which always has a greater value than the 3-month option; thus, the longer an option has until expiration, the greater its value. This is because stock prices move up on average, so a longer time until expiration means a greater chance for the option to be in-the-money by its expiration date, making the option more valuable.

Shown in the following table are the Black-Scholes model prices for Western's call option with the original inputs except for standard deviation, which is allowed to vary.

Standard Deviation (σ)	Call Option Price
0.001%	$ 6.37
10.000	6.22
31.557	7.39
40.000	8.72
60.000	11.91
90.000	16.37

The first row shows the option price if there is very little stock volatility.[17] Notice that as volatility increases, so does the option price. Therefore, the riskier the underlying security, the more valuable the option. To see why this makes sense, suppose you bought a call option with a strike price equal to the current stock price. If the stock had no risk (which means σ = 0), then there would be a zero probability of the stock going up, hence a zero probability of making money on the option. On the other hand, if you bought an option on a high-variance stock, there would be a higher probability that the stock would go way up and hence that you would make a large profit on the option. Of course, a high-variance stock could go way down, but as an option holder your losses would be limited to the price paid for the option—only the right-hand side of the stock's probability distribution counts. Put another way, an increase in the price of the stock helps option holders more than a decrease hurts them, so the greater the stock's volatility, the greater the value of the option. This makes options on risky stocks more valuable than those on safer, low-risk stocks. For example, an option on Cisco should have a greater value than an otherwise identical option on Kroger, the grocery store chain.

Shown below are the prices for Western's call option with the original inputs except for the risk-free rate, which is allowed to vary.

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[17]With such a low standard deviation, the current stock price of $40 is unlikely to change very much before expiration, so the option will be in-the-money at expiration and the owner will certainly pay the strike price and exercise the option at that time. This means that the present value of the strike price is the cost of exercising expressed in today's dollars. The present value of a stock's expected cash flows is equal to the current stock price. So the value of the option today is approximately equal to the current stock price of $40 less the present value of the strike price that must be paid when the stock is exercised at expiration. If we assume daily compounding, then the current option price should be:

$$V_C(\text{for } \sigma = 0.001\%) \approx \$40 - \frac{\$35}{\left(1 + \frac{0.08}{365}\right)^{365(0.5)}} = \$6.37$$

Observe that this is the same value given by the Black-Scholes model, even though we calculated it more directly. This approach only works if the volatility is almost zero.

Taxes and Stock Options

If an employee stock option grant meets certain conditions, it is called a "tax-qualifying grant" or sometimes an "Incentive Stock Option"; otherwise, it is a "nonqualifying grant." For example, suppose you receive a grant of 1,000 options with an exercise price of $50. If the stock price goes to $110 and you exercise the options, you must pay $50(1,000) = $50,000 for stock that is worth $110,000, which is a sweet deal. But what is your tax liability? If you receive a nonqualifying grant, then you are liable for ordinary income taxes on 1,000($110 − $50) = $60,000 when you exercise the option. But if it is a tax-qualified grant, you owe no regular taxes when exercised. By waiting at least a year and then selling the stock for, say, $150, you would have a long-term capital gain of 1,000($150 − $50) = $100,000, which would be taxed at the lower capital gains rate.

Before you gloat over your newfound wealth, you had better consult your accountant. Your "profit" when you exercise the tax-qualified options isn't taxable under the regular tax code, but it is under the Alternative Minimum Tax (AMT) code. With an AMT tax rate of up to 28%, you might owe as much as 0.28($110 − $50)(1,000) = $16,800. Here's where people get into trouble. The AMT tax isn't due until the following April, so you might think about waiting until then to sell some stock to pay your AMT tax (so that the sale will qualify as a long-term capital gain).

But what happens if the stock price falls to $5 by next April? You can sell your stock, which raises only $5(1,000) = $5,000 in cash. Without going into the details, you will have a long-term capital loss of 1,000 ($50 − $5) = $45,000 but IRS regulations limit your net capital loss in a single year to $3,000. In other words, the cash from the sale and the tax benefit from the capital loss aren't nearly enough to cover the AMT tax. You may be able to reduce your taxes in future years because of the AMT tax you pay this year and the carry-forward of the remaining long-term capital loss, but that doesn't help right now. You lost $45,000 of your original $50,000 investment, you now have very little cash, and—adding insult to injury—the IRS will insist that you also pay the $16,800 AMT tax.

This is exactly what happened to many people who made paper fortunes in the dot-com boom only to see them evaporate in the ensuing bust. They were left with worthless stock but multimillion-dollar AMT tax obligations. In fact, many still have IRS liens garnishing their wages until they eventually pay their AMT tax. So if you receive stock options, we congratulate you. But unless you want to be the next poster child for poor financial planning, we advise you to settle your AMT tax when you incur it.

Risk-Free Rate (r_{RF})	Call Option Price
0%	$6.41
4	6.89
8	7.39
12	7.90
20	8.93

As the risk-free rate increases, the value of the option increases. The principal effect of an increase in r_{RF} is to reduce the present value of the exercise price, which increases the current value of the option. Option prices in general are not very sensitive to interest rate changes, at least not to changes within the ranges normally encountered.

Myron Scholes and Robert Merton (who also was a pioneer in the field of options) were awarded the 1997 Nobel Prize in Economics, and Fischer Black would have been a co-recipient had he still been living. Their work provided analytical tools and methodologies that are widely used to solve many types of financial problems, not just option pricing. Indeed, the entire field of modern risk management is based primarily on their contributions. Although the Black-Scholes model was derived for a European option that can be exercised only on its maturity date, it also applies to

American options that don't pay any dividends prior to expiration. The textbooks by Don Chance and John Hull (cited in footnote 1) show adjusted models for dividend-paying stocks.

What is the purpose of the Black-Scholes option pricing model?

Explain what a "riskless hedge" is and how the riskless hedge concept is used in the Black-Scholes OPM.

Describe the effect of a change in each of the following factors on the value of a call option: (1) stock price, (2) exercise price, (3) option life, (4) risk-free rate, and (5) stock return standard deviation (i.e., risk of stock).

Using an *Excel* worksheet, what is the value of a call option with these data: P = $35, X = $25, r_{RF} = 6%, t = 0.5 (6 months), and σ = 0.6? **($12.05)**

8.6 THE VALUATION OF PUT OPTIONS

A put option gives its owner the right to sell a share of stock. If the stock pays no dividends and the option can be exercised only upon its expiration date, what is its value? Rather than reinventing the wheel, consider the payoffs for two portfolios at expiration date T, as shown in Table 8-2. The first portfolio consists of a put option and a share of stock; the second has a call option (with the same strike price and expiration date as the put option) and some cash. The amount of cash is equal to the present value of the exercise cost discounted at the continuously compounded risk-free rate, which is $Xe^{-r_{RF}t}$. At expiration, the value of this cash will equal the exercise cost, X.

If P_T, the stock price at expiration date T, is less than X, the strike price when the option expires, then the value of the put option at expiration is $X - P_T$. Therefore, the value of Portfolio 1, which contains the put and the stock, is equal to X minus P_T plus P_T, or just X. For Portfolio 2, the value of the call is zero at expiration (because the call option is out-of-the-money), and the value of the cash is X, for a total value of X. Notice that both portfolios have the same payoffs if the stock price is less than the strike price.

What if the stock price is greater than the strike price at expiration? In this case, the put is worth nothing, so the payoff of Portfolio 1 is equal to P_T, the stock price at expiration. The call option is worth $P_T - X$, and the cash is worth X, so the payoff of Portfolio 2 is P_T. Hence the payoffs of the two portfolios are equal regardless of whether the stock price is below or above the strike price.

If the two portfolios have identical payoffs, then they must have identical values. This is known as the **put–call parity relationship**:

TABLE 8-2	Portfolio Payoffs		
		PAYOFF AT EXPIRATION IF:	
		$P_T < X$	$P_T \geq X$
Put		$X - P_T$	0
Stock		P_T	P_T
	Portfolio 1:	X	P_T
Call		0	$P_T - X$
Cash		X	X
	Portfolio 2:	X	P_T

Put option + Stock = Call option + PV of exercise price.

If V_c is the Black-Scholes value of the call option, then the value of a put is[18]

$$\text{Put option} = V_C - P + Xe^{-r_{RF}t} \qquad \text{(8-12)}$$

For example, consider a put option written on the stock discussed in the previous section. If the put option has the same exercise price and expiration date as the call, then its price is

$$\text{Put option} = \$7.39 - \$40 + \$35e^{-0.08(0.5)}$$
$$= \$7.39 - \$40 + \$33.63 = \$1.02$$

It is also possible to modify the Black-Scholes call option formula to obtain a put option formula:

$$\text{Put option} = P[N(d_1) - 1] - Xe^{-r_{RF}t}[N(d_2) - 1] \qquad \text{(8-13)}$$

The only difference between this formula for puts and the formula for calls is the subtraction of 1 from $N(d_1)$ and $N(d_2)$ in the call option formula.

Self-Test

In words, what is put–call parity?

A put option written on the stock of Taylor Enterprises (TE) has an exercise price of $25 and 6 months remaining until expiration. The risk-free rate is 6%. A call option written on TE has the same exercise price and expiration date as the put option. TE's stock price is $35. If the call option has a price of $12.05, then what is the price (i.e., value) of the put option? **($1.31)**

8.7 Applications of Option Pricing in Corporate Finance

Option pricing is used in four major areas of corporate finance: (1) real options analysis for project evaluation and strategic decisions, (2) risk management, (3) capital structure decisions, and (4) compensation plans.

Real Options

Suppose a company has a 1-year proprietary license to develop a software application for use in a new generation of wireless cellular telephones. Hiring programmers and marketing consultants to complete the project will cost $30 million. The good news is that if consumers love the new cell phones, there will be a tremendous demand for the software. The bad news is that if sales of the new cell phones are low, the software project will be a disaster. Should the company spend the $30 million and develop the software?

Because the company has a license, it has the option of waiting for a year, at which time it might have a much better insight into market demand for the new cell phones. If demand is high in a year, then the company can spend the $30 million and develop the software. If demand is low, it can avoid losing the $30 million devel-

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[18]This model cannot be applied to an American put option or to a European option on a stock that pays a dividend prior to expiration. For an explanation of valuation approaches in these situations, see the books by Chance and Hull cited in footnote 1.

opment cost by simply letting the license expire. Notice that the license is analogous to a call option: It gives the company the right to buy something (in this case, software for the new cell phones) at a fixed price ($30 million) at any time during the next year. The license gives the company a **real option**, because the underlying asset (the software) is a real asset and not a financial asset.

There are many other types of real options, including the option to increase capacity at a plant, to expand into new geographical regions, to introduce new products, to switch inputs (such as gas versus oil), to switch outputs (such as producing sedans versus SUVs), and to abandon a project. Many companies now evaluate real options with techniques that are similar to those described earlier in the chapter for pricing financial options.

Risk Management

Suppose a company plans to issue $400 million of bonds in 6 months to pay for a new plant now under construction. The plant will be profitable if interest rates remain at current levels, but if rates rise then it will be unprofitable. To hedge against rising rates, the company could purchase a put option on Treasury bonds. If interest rates go up then the company would "lose" because its bonds would carry a high interest rate, but it would have an offsetting gain on its put options. Conversely, if rates fall then the company would "win" when it issues its own low-rate bonds, but it would lose on the put options. By purchasing puts, the company has hedged the risk due to possible interest rate changes that it would otherwise face.

Another example of risk management is a firm that bids on a foreign contract. For example, suppose a winning bid means that the firm will receive a payment of 12 million euros in 9 months. At a current exchange rate of $1.57 per euro, the project would be profitable. But if the exchange rate falls to $1.10 per euro, the project would be a loser. To avoid exchange rate risk, the firm could take a short position in a forward contract that allows it to convert 12 million euros into dollars at a fixed rate of $1.50 per euro in 9 months, which would still ensure a profitable project. This eliminates exchange rate risk if the firm wins the contract, but what if the firm loses the contract? It would still be obligated to sell 12 million euros at a price of $1.50 per euro, which could be a disaster. For example, if the exchange rate rises to $1.75 per euro, then the firm would have to spend $21 million to purchase 12 million euros at a price of $1.75/€ and then sell the euros for $18 million = ($1.50/€)(€12 million), a loss of $3 million.

To eliminate this risk, the firm could instead purchase a currency put option that allows it to sell 12 million euros in 9 months at a fixed price of $1.50 per euro. If the company wins the bid, it will exercise the put option and sell the 12 million euros for $1.50 per euro if the exchange rate has declined. If the exchange rate hasn't declined, then it will sell the euros on the open market for more than $1.50 and let the option expire. On the other hand, if the firm loses the bid, then it has no reason to sell euros and could let the option contract expire. Note, however, that even if the firm doesn't win the contract, it still is gambling on the exchange rate because it owns the put; if the price of euros declines below $1.50, the firm will still make some money on the option. Thus, the company can lock in the future exchange rate if it wins the bid and can avoid any net payment at all if it loses the bid. The total cost in either scenario is equal to the initial cost of the option. In other words, the cost of the option is like insurance that guarantees the exchange rate if the company wins the bid and guarantees no net obligations if it loses the bid.

Many other applications of risk management involve futures contracts and other complex derivatives rather than calls and puts. However, the principles used in

pricing derivatives are similar to those used earlier in this chapter for pricing options. Thus, financial options and their valuation techniques play key roles in risk management.

Capital Structure Decisions

Decisions regarding the mix of debt and equity used to finance operations are quite important. One interesting aspect of the capital structure decision is based on option pricing. For example, consider a firm with debt requiring a final principal payment of $60 million in 1 year. If the company's value 1 year from now is $61 million, then it can pay off the debt and have $1 million left for stockholders. If the firm's value is less than $60 million, then it may well file for bankruptcy and turn over its assets to creditors, resulting in stockholders' equity of zero. In other words, the value of the stockholders' equity is analogous to a call option: The equity holders have the right to buy the assets for $60 million (which is the face value of the debt) in 1 year (when the debt matures).

Suppose the firm's owner-managers are considering two projects. One project has very little risk, and it will result in an asset value of either $59 million or $61 million. The other has high risk, and it will result in an asset value of either $20 million or $100 million. Notice that the equity will be worth zero if the assets are worth less than $60 million, so the stockholders will be no worse off if the assets end up at $20 million than if they end up at $59 million. On the other hand, the stockholders would benefit much more if the assets were worth $100 million than $61 million. Thus, the owner-managers have an incentive to choose risky projects, which is consistent with an option's value rising with the risk of the underlying asset. Potential lenders recognize this situation, so they build covenants into loan agreements that restrict managers from making excessively risky investments.

Not only does option pricing theory help explain why managers might want to choose risky projects (consider, for example, the case of Enron) and why debtholders might want restrictive covenants, but options also play a direct role in capital structure choices. For example, a firm could choose to issue convertible debt, which gives bondholders the option to convert their debt into stock if the value of the company turns out to be higher than expected. In exchange for this option, bondholders charge a lower interest rate than for nonconvertible debt. Because owner-managers must share the wealth with convertible-bond holders, they have a smaller incentive to gamble with high-risk projects.

Compensation Plans

Many companies use stock options as a part of their compensation plans. It is important for boards of directors to understand the value of these options before they grant them to employees. We discuss compensation issues associated with stock options in more detail in Chapter 13.

Self-Test   Describe four ways that option pricing is used in corporate finance.

Summary

In this chapter we discussed option pricing topics, which included the following.

- **Financial options** are instruments that (1) are created by exchanges rather than firms, (2) are bought and sold primarily by investors, and (3) are of importance to both investors and financial managers.

- The two primary types of financial options are (1) **call options**, which give the holder the right to purchase a specified asset at a given price (the **exercise**, or **strike, price**) for a given period of time, and (2) **put options**, which give the holder the right to sell an asset at a given price for a given period of time.
- A call option's **exercise value** is defined as the maximum of zero or the current price of the stock less the strike price.
- The **Black-Scholes option pricing model (OPM)** or the **binomial model** can be used to estimate the value of a call option.
- The five inputs to the Black-Scholes model are (1) P, the current stock price; (2) X, the strike price; (3) r_{RF}, the risk-free interest rate; (4) t, the remaining time until expiration; and (5) σ, the standard deviation of the stock's rate of return.
- A call option's value increases if P increases, X decreases, r_{RF} increases, t increases, or σ increases.
- The **put–call parity relationship** states that

$$\text{Put option} + \text{Stock} = \text{Call option} + \text{PV of exercise price.}$$

Questions

(8–1)   Define each of the following terms:
  a.   Option; call option; put option
  a.   Exercise value; strike price
  c.   Black-Scholes option pricing model

(8–2)   Why do options sell at prices higher than their exercise values?

(8–3)   Describe the effect on a call option's price that results from an increase in each of the following factors: (1) stock price, (2) strike price, (3) time to expiration, (4) risk-free rate, and (5) standard deviation of stock return.

Self-Test Problems   Solutions Appear in Appendix A

(ST–1)
Binomial Option
Pricing
The current price of a stock is $40. In 1 year, the price will be either $60 or $30. The annual risk-free rate is 5%. Find the price of a call option on the stock that has an exercise price of $42 and that expires in 1 year. (*Hint:* Use daily compounding.)

(ST–2)
Black-Scholes
Model
Use the Black-Scholes Model to find the price for a call option with the following inputs: (1) current stock price is $22, (2) strike price is $20, (3) time to expiration is 6 months, (4) annualized risk-free rate is 5%, and (5) standard deviation of stock return is 0.7.

Problems   Answers Appear in Appendix B

EASY PROBLEMS 1–2

(8–1)
Options
A call option on the stock of Bedrock Boulders has a market price of $7. The stock sells for $30 a share, and the option has a strike price of $25 a share. What is the exercise value of the call option? What is the option's time value?

(8–2)
Options

The exercise price on one of Flanagan Company's options is $15, its exercise value is $22, and its time value is $5. What are the option's market value and the price of the stock?

(8–3)
Black-Scholes Model

Assume that you have been given the following information on Purcell Industries:

Current stock price = $15	Strike price of option = $15
Time to maturity of option = 6 months	Risk-free rate = 6%
Variance of stock return = 0.12	
$d_1 = 0.24495$	$N(d_1) = 0.59675$
$d_2 = 0.00000$	$N(d_2) = 0.50000$

According to the Black-Scholes option pricing model, what is the option's value?

(8–4)
Put–Call Parity

The current price of a stock is $33, and the annual risk-free rate is 6%. A call option with a strike price of $32 and with 1 year until expiration has a current value of $6.56. What is the value of a put option written on the stock with the same exercise price and expiration date as the call option?

(8–5)
Black-Scholes Model

Use the Black-Scholes Model to find the price for a call option with the following inputs: (1) current stock price is $30, (2) strike price is $35, (3) time to expiration is 4 months, (4) annualized risk-free rate is 5%, and (5) variance of stock return is 0.25.

(8–6)
Binomial Model

The current price of a stock is $20. In 1 year, the price will be either $26 or $16. The annual risk-free rate is 5%. Find the price of a call option on the stock that has a strike price of $21 and that expires in 1 year. (*Hint:* Use daily compounding.)

(8–7)
Binomial Model

The current price of a stock is $15. In 6 months, the price will be either $18 or $13. The annual risk-free rate is 6%. Find the price of a call option on the stock that has a strike price of $14 and that expires in 6 months. (*Hint:* Use daily compounding.)

SPREADSHEET PROBLEM

(8-8)
Build a Model: Black-Scholes Model

resource

Start with the partial model in the file *Ch08 P08 Build a Model.xls* on the textbook's Web site. You have been given the following information for a call option on the stock of Puckett Industries: P = $65.00, X = $70.00, t = 0.50, r_{RF} = 5.00% and σ = 50.00%.
a. Use the Black-Scholes option pricing model to determine the value of the call option.
b. Suppose there is a put option on Puckett's stock with exactly the same inputs as the call option. What is the value of the put?

Mini Case

Assume that you have just been hired as a financial analyst by Triple Play Inc., a mid-sized California company that specializes in creating high-fashion clothing. Because no one at Triple Play is familiar with the basics of financial options, you have been asked to prepare a brief report that the firm's executives can use to gain at least a cursory understanding of the topic.

To begin, you gathered some outside materials on the subject and used these materials to draft a list of pertinent questions that need to be answered. In fact, one possible approach to the report is to use a question-and-answer format. Now that the questions have been drafted, you have to develop the answers.

a. What is a financial option? What is the single most important characteristic of an option?

b. Options have a unique set of terminology. Define the following terms:

    (1) Call option
    (2) Put option
    (3) Strike price or exercise price
    (4) Expiration date
    (5) Exercise value
    (6) Option price
    (7) Time value
    (8) Writing an option
    (9) Covered option
    (10) Naked option
    (11) In-the-money call
    (12) Out-of-the-money call
    (13) LEAPS

c. Consider Triple Play's call option with a $25 strike price. The following table contains historical values for this option at different stock prices:

Stock Price	Call Option Price
$25	$ 3.00
30	7.50
35	12.00
40	16.50
45	21.00
50	25.50

    (1) Create a table that shows (a) stock price, (b) strike price, (c) exercise value, (d) option price, and (e) the time value, which is the option's price less its exercise value.
    (2) What happens to the time value as the stock price rises? Why?

d. Consider a stock with a current price of P = $27. Suppose that over the next 6 months the stock price will either go up by a factor of 1.41 or down by a factor of 0.71. Consider a call option on the stock with a strike price of $25 that expires in 6 months. The risk-free rate is 6%.

    (1) Using the binomial model, what are the ending values of the stock price? What are the payoffs of the call option?
    (2) Suppose you write 1 call option and buy N_s shares of stock. How many shares must you buy to create a portfolio with a riskless payoff (i.e., a hedge portfolio)? What is the payoff of the portfolio?
    (3) What is the present value of the hedge portfolio? What is the value of the call option?
    (4) What is a replicating portfolio? What is arbitrage?

e. In 1973, Fischer Black and Myron Scholes developed the Black-Scholes option pricing model (OPM).

(1) What assumptions underlie the OPM?
(2) Write out the three equations that constitute the model.
(3) According to the OPM, what is the value of a call option with the following characteristics?

Stock price = $27.00
Strike price = $25.00
Time to expiration = 6 months = 0.5 years
Risk-free rate = 6.0%
Stock return standard deviation = 0.49

f. What impact does each of the following parameters have on the value of a call option?

(1) Current stock price
(2) Strike price
(3) Option's term to maturity
(4) Risk-free rate
(5) Variability of the stock price

g. What is put–call parity?